Abstract. In general, the difficulty to characterise non-equilibrium systems lies in the fact that there is no analog of the Boltzmann-distribution to describe thermodynamic variables and their fluctuations. Over the last 20 years, however, it was observed that there is a class of classical non-equilibrium systems with diffusive transport in which the statistics of density and current profiles show universal properties that do not depend on the microscopic details of the model. The general framework to characterise these systems from a macroscopic point of view is today called the "Macroscopic Fluctuation Theory". A natural question is whether this framework can be extended to quantum mechanics to describe the statistics of purely quantum mechanical effects such as entanglement in diffusive out-of-equilibrium systems. With this aim in mind, I will introduce the Quantum Simple Symmetric Exclusion Process (Q-SSEP), a microscopic toy model, from which we hope to gain insights in possible universal features of these quantum mechanical effects. I will present the results obtained so far and comment shortly on the recent observation that free cumulants, a tool from free probability theory, seems to play a role in the mathematical structure of the model.